

Development of Two Probability Distributions to Analyze Impact Injury Data

Arnold K. Johnson

U S. Department of Transportation
National Highway Traffic Safety Administration
400 Seventh Street, S. W.
Washington, DC 20590

ABSTRACT

Impact injury data are not, in general, suitable for statistical analysis by least squares, because the data are not observations from normal probability distributions. The principle of maximum likelihood can be used as an alternative, but requires that the appropriate probability distribution be specified. Two appropriate distributions are developed, one for thoracic fractures and another for the Abbreviated Injury Scale. A data sample for each of these two types of data is analyzed by the principle of maximum likelihood in conjunction with the appropriate probability distribution.

THE METHOD OF LEAST SQUARES is often used to analyze injury data. This method is suitable for analyzing data which are observations from a family of normal, bell-shaped (Gaussian) probability distributions such as those of Figure 1. The random variable ranges from negative infinity to positive infinity, and each point on the regression line is an estimate of the mean of a normal distribution. Least squares provides an estimate of the regression line and the standard deviation.

Impact injury to cadavers is usually assessed in terms of the Abbreviated Injury Scale (AIS) (1), or, for impacts to the thorax, in terms of the number of thoracic fractures. Neither the observed AIS nor the observed number of thoracic fractures is an observation from a normal distribution. First, both AIS scores and thoracic fractures are equal to or greater than zero, never negative. Second, they are not in general observations from a bell-shaped distribution, like those in Figure 1. Consider first thoracic fractures.

For a very mild impact shown in Figure 2, all the probability of unity is at zero fractures. For such a mild impact, it is almost a

certainty that a fracture will not occur. For a more severe impact shown in Figure 3, the probability of zero fractures might drop to 0.7 with the remaining 0.3 probability distributed as shown. This distribution is not bell-shaped. As the impact severity increases, the probability of zero fractures decreases with the bulk of the probability moving towards a greater number of fractures. For a very severe impact, as in Figure 4, there is essentially no probability of zero fractures, and the distribution is considerably removed from zero and is assumed to be approximately normal. The general features in Figures 2 and 3 are realistic. This paper assumes for severe impacts that the distribution is approximately normal. More will be said later about the assumed probability distribution for thoracic fractures.

Consider next AIS probability distributions. The probability distribution for thoracic fractures is a phenomenon of nature. However, the probability distribution for the AIS depends not on nature, but on what is written in the AIS handbook (1). For fractures, the distribution can be expected to vary smoothly as shown in Figure 4. No such variation can be expected for AIS data which have only seven possible outcomes from zero to six inclusive. The probabilities could make large positive or negative changes from one AIS category to the next. Figure 5 is a fictitious AIS probability distribution which illustrates large positive and negative changes. Although one might assume that a large number of fractures is an observation from an approximately normal distribution as in Figure 4, it is difficult to assume, because of the limited number of outcomes, that a set of AIS data are all observations from approximately normal distributions.

* Numbers in parentheses designate references at the end of the paper.



Figure 1: A family of normal distribution curves which have as their mean the linear equation shown and for which observations from them are suitable for analysis by the method of least squares.

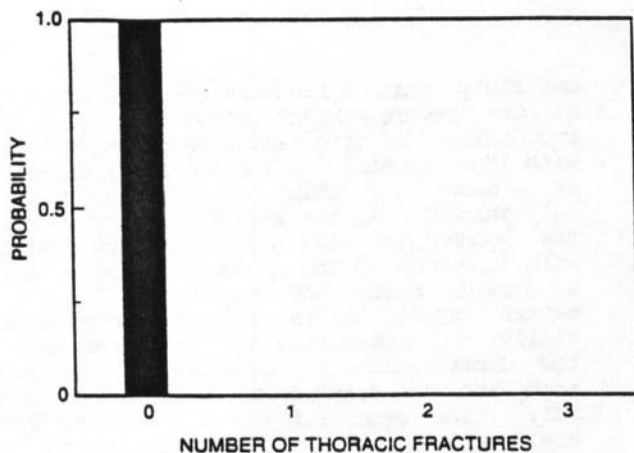


Figure 2: Probability distribution of thoracic fractures for an extremely mild impact with all the probability at zero fractures (i.e., no injury).

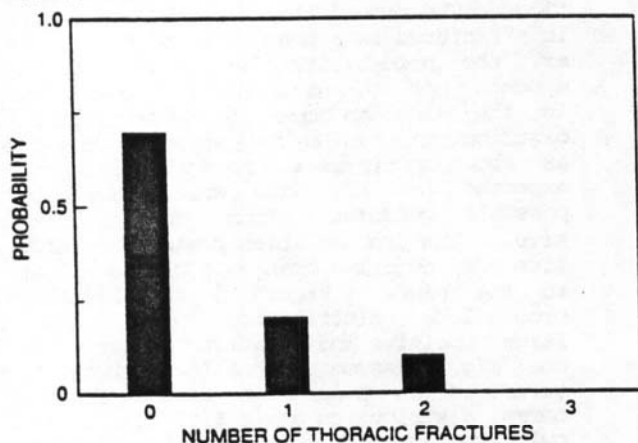


Figure 3: A fictitious probability distribution one might expect of thoracic fractures for a not too severe impact. (The distribution is not bell-shaped.)

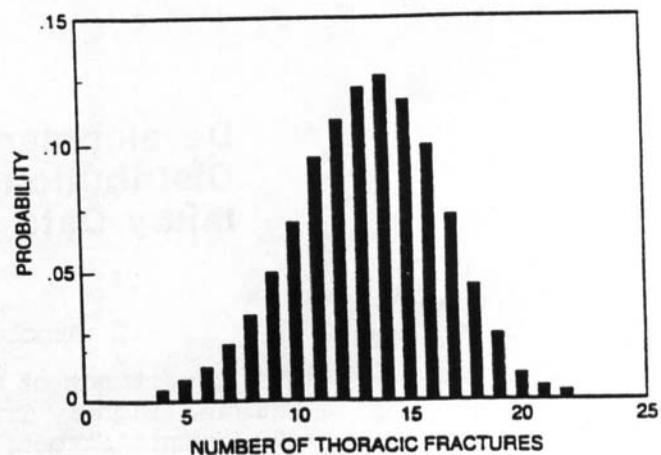


Figure 4: An assumed probability distribution of thoracic fractures for a severe impact. (The distribution is approximately bell-shaped.)

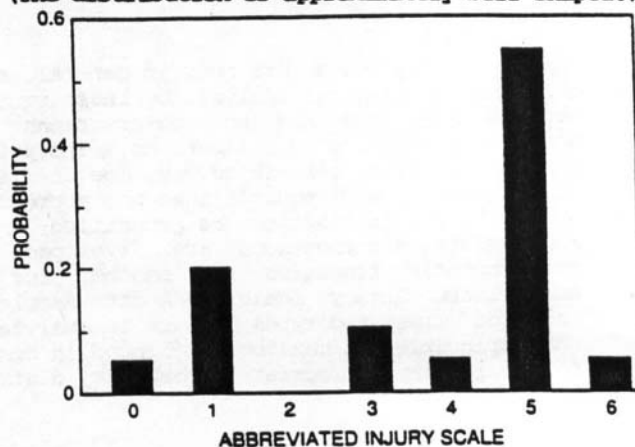


Figure 5: A fictitious probability distribution of AIS to illustrate how the probability might vary positively and negatively from one AIS category to the next. (Note that the probability for an AIS 2 is shown as zero.)

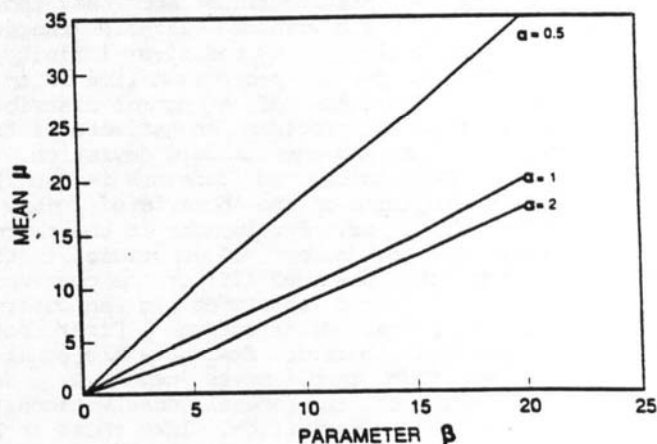


Figure 6: Plots of the mean μ of Equation 3 for various values of α as a function of the parameter β .

The method of least squares minimizes the sum of the deviations squared about an assumed mean, and it can be implemented for any set of data. However, the accuracy of least squares from a family of non-normal distributions would be in doubt.

An alternative to least squares is based on the statistical principle of maximum likelihood (PML). As discussed in (2), chapter 8, the PML is perhaps the best statistical procedure for estimating parameters in probability distributions. For a family of normal distributions as in Figure 1, the PML and least squares arrive at identical results. Least squares is a special case of the PML.

Biomechanics papers began appearing about 1984 concerning the PML. Examples are in (3), (4) and (5). Most of the papers concerned probit, Weibull and logistic analyses, in which the PML was used to estimate parameters. The analyses in general concerned probability distributions for which there were only two outcomes. For example, the distributions would often give the probability of injury or no injury. An example can be found in (3). In contrast, this present paper develops probability distributions for which there is a range of injury severity outcomes, i.e., the number of thoracic fractures or the AIS scores.

As stated, the above cited three methods are for the analysis of observations for which there are only two outcomes, and for this reason they have generally been intended for failure analysis or threshold studies. The probability distributions in this present paper provide a higher level of detail in regard to the probability of injury than do the above methods.

REVIEW OF THE PRINCIPLE OF MAXIMUM LIKELIHOOD

This is a brief outline of the PML. More details can be found in (2) and other standard statistics textbooks.

Each item in a data sample is a random variable, and thus the sample itself is a random variable. Sampling theory assumes that one is lucky and has observed a sample which has a high probability of being observed. The PML assumes the ultimate, that the observed sample has the highest probability of all samples. It uses this assumption to estimate parameters in a probability distribution similar to estimating the parameters of a normal distribution by least squares. However, instead of minimizing a function, as in least squares, a function is maximized. The function is now described.

If the probability of observing any outcome in a sample is independent of all the other observed outcomes, then the observations are said to be independent in the probability sense. The probability distribution for such a sample is the product of the probabilities of each item in the sample. For example, let

$$P(k; \rho)$$

be the probability of observing the outcome k , where ρ represents the set of parameters to be estimated by the PML. For a sample of size N , we have the following set of observations,

$$k_1, k_2, \dots, k_N$$

and

$$P(k_1; \rho), P(k_2; \rho), \dots, P(k_N; \rho)$$

are their corresponding probabilities. The probability distribution of the sample, usually called L in the theory of the PML, is the product of these probabilities:

$$L = P(k_1; \rho) P(k_2; \rho) \dots P(k_N; \rho) \quad \text{Eq. (1)}$$

In summary, the function L is the probability distribution for a set of observations which are independent in the probability sense. The PML requires selecting that set of values of the parameters represented by ρ which maximizes L . Maximizing L , called the likelihood function, is the essence of the PML.

For this report, a numerical procedure to maximize L was incorporated into the computer program used for the analyses presented. (The program and its documentation can be obtained from the author.)

In order to implement the PML, it is necessary to assume the true distribution of the data. To the extent that an assumed distribution is in error, an error is introduced into the analysis. Faith in the results rests upon the logic used to establish a probability distribution and the reasonableness of the results. The distribution for fractures will be presented first and that for the AIS later.

THE DEVELOPMENT OF A PROBABILITY DISTRIBUTION FOR THORACIC FRACTURES

There are no known data by which to determine the features of the probability distribution of thoracic fractures. Nor does it appear feasible how such data could be generated because of the wide variability of cadavers. Therefore, in the absence of such data, appropriate features will be assumed based on common sense, and then a probability distribution with those features will be presented.

The following features are considered appropriate for a probability distribution for thoracic fractures:

- (1) The possible outcomes are integers ranging from zero to positive infinity.
- (2) The distribution is not erratic. That is, as one starts at zero and goes to a higher number of fractures, there is a smooth change in the probabilities.
- (3) For an impact of sufficient severity such that the probability of no injury is small, the distribution's envelope approximates the curve of a normal probability distribution similar to what is shown in Figure 4. This

assumption is based on the fact that many observables in nature have normal distributions.

(4) The probability of no thoracic fractures approaches unity (i.e., the probability of no thoracic fractures approaches certainty) as the impact severity approaches zero.

(5) Once the probabilities decline as one moves to higher numbers of fractures, the probabilities continue to decline. (In practical terms, this means that the probability distribution cannot have a local minimum, something which is difficult to imagine.)

A probability distribution that possesses the above features is

$$P(k) = e^{-(k/\beta)^{\alpha}} - e^{-((k+1)/\beta)^{\alpha}} \quad \text{Eq. (2)}$$

where $P(k)$ is the probability of observing exactly k fractures, and it was shown by evaluating Eq. (2) for various pairs of values of the parameters α and β that the distribution has the features described above.

The development of a probability distribution for fractures also requires establishing how best to model the distribution's two parameters α and β as functions of the experimental data. The first problem was to establish how α and β change with impact severity. It was easy to analyze Eq. (2) to show that β equal to zero is the condition for $P(0)=1$ (no impact). Thus, β as a function of an impact severity parameter (e.g., maximum tensile force in a belt) passes through the origin. The behavior of β away from the origin was established by studying the mean of the probability distribution of Eq. (2).

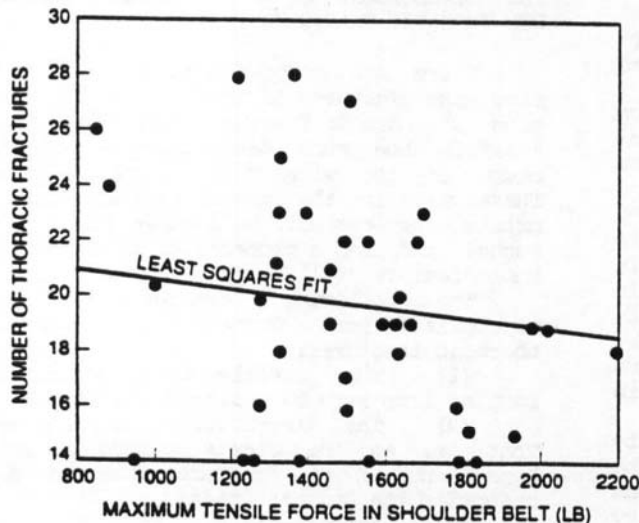


Figure 7: A least squares fit for a large number of thoracic fractures versus tensile force in a shoulder belt for cadavers restrained by a lap and shoulder belt in forward facing impacts. (The number of fractures is restricted to fourteen or more.)

The mean of the probability distribution of Eq. (2) is

$$\mu = \sum_{k=0}^{\infty} k [e^{-(k/\beta)^{\alpha}} - e^{-((k+1)/\beta)^{\alpha}}] \quad \text{Eq. (3)}$$

The curves in Figure 6 show that μ is approximately directly proportional to β with the proportionality constant a function of α . Knowledge of the mean of thoracic fractures as a function of impact severity would provide direct knowledge on how best to model β , because the two parameters are directly proportional to each other.

Table 1 (6) has data for 106 cadavers restrained by a lap and shoulder belt in forward facing impacts. The number of thoracic fractures is given along with the maximum tensile force (MTF) in the shoulder belt, the age and the weight of the cadaver. Although, in general, thoracic fractures are not observations from a normal distribution, this present paper assumes that a large number of fractures is an observation from an approximately normal distribution as shown in Figure 4. Thus, an analysis by least squares to estimate the mean of a sample for a large number of fractures is assumed valid for this special case. Figure 7 shows a plot from Table 1 of fractures versus MTF together with a least squares fit to a straight line for fourteen or more fractures as a function of the MTF.

Figure 7 shows that the mean number of fractures in the range of severe impacts is not dependent on the MTF. A plateau, or better stated, a saturation is attained. It appears that there is a maximum number of fractures that can occur on average. (The effect that increased impact severity might have on organs within the thorax is not considered in this analysis of fractures.) No significance is attached to the line's slightly negative slope. The true line, for which the line shown is an estimate, is assumed to be horizontal. Since β is proportional to the mean μ and the mean becomes horizontal for severe impacts, so should β .

For a region of very mild impacts there is little probability of injury, and the mean, together with β , would essentially be horizontal, very close to zero. Thus, β should be horizontal in a region of very mild impacts and become horizontal again in the region of severe impacts. Between these two horizontal regions the mean, together with β , gradually increases with impact severity. This paper assumes that β has the "S" curve shown in Figure 8 of the form

$$\beta = \beta_0 (1 - e^{-f(X)}) \quad \text{Eq. (4)}$$

where $f(X) \geq 0$ and X is a parameter which specifies impact severity.

The modeling of α was accomplished, as for β , to assure that the mean μ increases

TABLE 1 — THORACIC FRACTURE DATA (6)

WT (LB)	AGE (YR)	FORCE (LB)	FRACTURES	WT (LB)	AGE (YR)	FORCE (LB)	FRACTURES
143	33	1621	6	163	22	1293	0
114	12	991	2	174	32	1553	5
97	70	881	24	116	34	982	1
103	73	926	13	178	48	1168	5
79	57	1101	10	114	50	935	12
150	59	1389	23	148	55	928	10
97	82	1214	28	141	56	989	10
190	39	1767	5	136	37	1003	3
99	71	1101	18	136	76	843	26
183	32	1677	9	178	45	1054	8
200	75	1632	19	129	58	922	14
154	62	1677	22	117	62	1753	13
192	53	1632	20	156	52	2203	13
145	40	1677	19	154	60	989	9
183	39	1742	8	187	50	1573	6
154	34	1544	14	132	57	1473	11
190	45	1720	10	121	60	1101	18
120	37	1434	12	154	33	1596	10
117	52	1333	18	154	53	1506	22
192	65	1499	17	154	57	775	7
179	45	1587	19	163	57	1978	19
152	65	1324	25	132	43	1910	13
154	36	1280	16	106	55	1506	27
132	35	1499	5	136	57	1461	19
123	22	1544	4	139	48	1416	11
172	49	1807	15	154	53	1303	10
172	33	1564	9	110	51	472	0
143	67	1544	22	150	57	1169	13
167	42	1697	12	132	59	854	11
123	19	1412	2	189	60	944	2
176	61	1499	9	110	60	831	2
126	55	1454	13	145	53	1258	13
134	75	1454	21	139	64	1236	14
152	30	1632	4	139	46	966	2
110	44	1499	7	132	34	899	0
132	79	1366	28	182	53	560	0
132	16	1477	1	122	58	710	1
187	59	1697	23	153	41	1140	0
139	22	1544	10	148	57	1020	7
154	25	1544	7	154	32	1560	0
165	58	1389	14	226	56	1790	14
134	38	1059	9	158	50	1930	14
108	54	1059	12	192	61	1930	15
205	24	1598	1	216	58	1650	11
125	24	1256	6	137	61	1595	11
152	35	1256	6	179	64	1495	12
126	55	1258	14	122	52	1325	9
106	21	1258	3	135	46	1781	16
198	26	1648	13	156	74	1250	12
163	22	1551	3	115	69	1330	23
132	38	888	2	162	44	2200	18
123	38	1223	7	125	65	2050	32
114	38	818	13	174	51	1200	8

as the impact severity increases. The parameter β by Eq.(4) was substituted into Eq.(3), and then μ was differentiated with respect to X . The derivative indicated that α modeled independent of X would assure that the mean would increase with impact severity. (In respect to Table 1, α could be modeled as a function of age and/or weight but not MTF, the impact severity parameter.)

The basic requirements for the modeling of α and β have been described. Modeling on a trial and error basis was still required. The model with the largest value of L (the figure of merit) was chosen as the best.

THE PROBABILITY OF THORACIC FRACTURES OF BELT RESTRAINED CADAVERS

The data in Table 1 (6) were analyzed by the FML to estimate the probability of thoracic fractures for belt restrained cadavers in forward facing impacts. Of the models investigated by trial and error, the best was for

$$\alpha = 1 + \rho_1 A$$

and

$$\beta = \rho_2 (A/W) (1 - e^{-\rho_3 (MTF)})^{\rho_4}$$

where A is age, W is weight, and MTF is the maximum tensile force in the shoulder belt. The ρ 's were determined by the FML to be

$$\begin{aligned} \rho_1 &= .023793 & \rho_3 &= 0.00386297 \\ \rho_2 &= 48.1592 & \rho_4 &= 19.2897 \end{aligned}$$

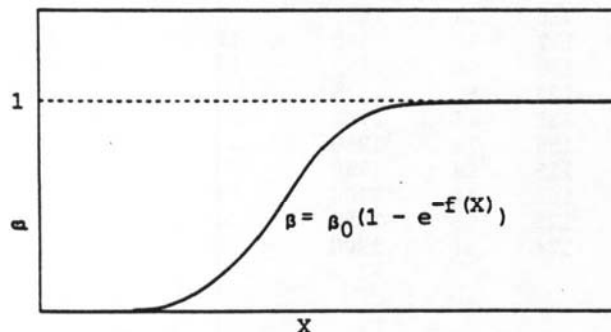


Figure 8: The "S" shaped curve assumed for β in Equation 2, as a function of X , a parameter which specifies impact severity.

Figure 9 is a bar graph of the probability (based on the above equations for α and β and the FML values) of thoracic fractures for a cadaver of weight 160 pounds, age 35 years and a MTF of 1200 pounds. Figure 10 contains the envelope of probability distributions for a cadaver of weight 160 pounds and MTF 1200 pounds to show the distributions as a function of age. Figure 11 shows the mean of three of the distributions in Figure 10 as a function of the MTF. In accordance with the modeling used, the means attain a plateau. (Note that although the probability distribution of Eq.(2) has two parameters, four parameters were used.)

THE PROBABILITY DISTRIBUTION FOR THE AIS

As discussed, the probability distribution for AIS is determined by what is written in the AIS handbook (1). The distribution chosen has

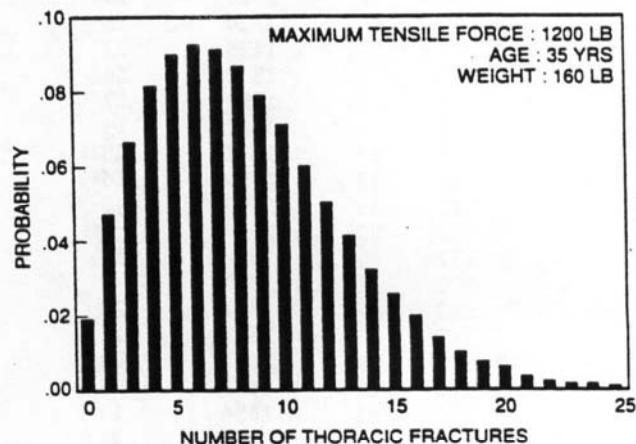


Figure 9: The probability distribution of thoracic fractures for a cadaver restrained by a lap and shoulder belt for the following conditions: weight 160 lb, age 35 years and the maximum tensile force in the belt 1200 lb.

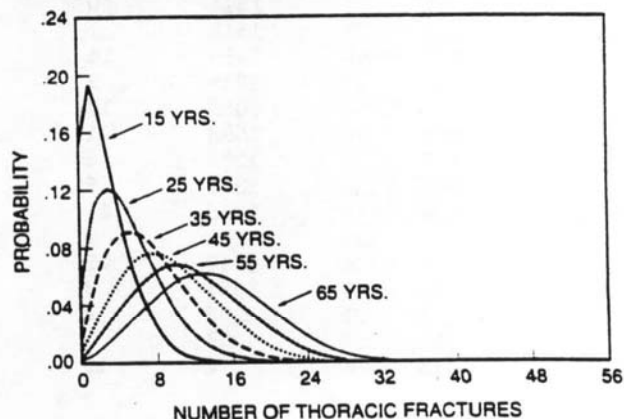


Figure 10: Probability distributions of thoracic fractures as a function of age for a cadaver restrained by a lap and shoulder belt for the following conditions: weight 160 lb and maximum tensile force in the shoulder belt 1200 lb.

to permit large positive and negative changes in the probabilities from one AIS category to the next. Consider the following distribution where each $\theta \geq 0$:

$$\begin{aligned} P(0) &= 1 - e^{-\theta_1} \\ P(1) &= e^{-\theta_1} - e^{-\theta_2} & P(4) &= e^{-\theta_4} - e^{-\theta_5} \\ P(2) &= e^{-\theta_2} - e^{-\theta_3} & P(5) &= e^{-\theta_5} - e^{-\theta_6} \\ P(3) &= e^{-\theta_3} - e^{-\theta_4} & P(6) &= e^{-\theta_6} \end{aligned} \quad \text{Eqs (5)}$$

A probability distribution must total one over all possible outcomes, and this distribution does so. Furthermore, all probabilities have to be positive numbers between zero and one. Reference is made to Figure 12 which is a graph of the function

$$e^{-\theta}$$

which varies from zero to one. Thus, the differences in Eqs.(5) are between zero and one provided that $\theta_{k+1} \geq \theta_k$. The probabilities can have large positive and negative changes from one AIS category to the next depending on where each θ is located. The PML determined the location of each.

A drawback of this AIS distribution is that it contains six parameters, the θ 's, whereas the distribution for fractures contains only α and β . The minimum number of PML parameters that have to be determined equals the number of parameters in the distribution. More can be added as in the analysis of fractures. For every parameter, there is one less degree of freedom in the statistical analysis.

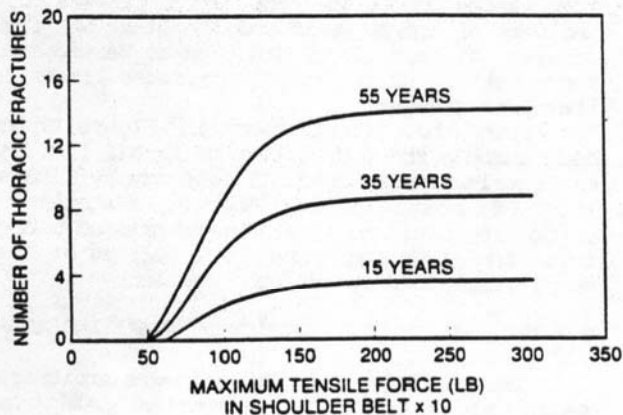


Figure 11: The mean of thoracic fractures of a 160 lb occupant as a function of the maximum tensile force in the shoulder belt for ages 15, 35, and 55 years.

The θ 's of Eqs.(5) were modeled by a simple scheme based on a factor $F \geq 0$ which was a function of cadaver age, weight and normalized compression (NC) of the thorax. The scheme was as follows:

$$\begin{aligned} \theta_1 &= \nu_1 F & \theta_4 &= \theta_3 + \nu_4 F \\ \theta_2 &= \theta_1 + \nu_2 F & \theta_5 &= \theta_4 + \nu_5 F \\ \theta_3 &= \theta_2 + \nu_3 F & \theta_6 &= \theta_5 + \nu_6 F \end{aligned} \quad \text{Eqs. (6)}$$

Each ν was positive and estimated by the PML. The scheme assumed that each successive θ would be larger than the preceding one, a requirement so that all the probabilities would be positive.

It was through the factor F that all modeling of the experimental data was accomplished. Although F could be any arbitrary function which was always positive, for this paper the normalized compression (NC) was always a factor in the denominator of F . As the severity of the impact becomes less and less severe and NC approaches zero, F and θ_1 approach infinity. As can be seen by Figure 12, this causes $P(0)$ to approach unity, as would be expected.

The mean of the distribution given by Eqs.(5) is

$$\begin{aligned} \mu &= \sum_{k=0}^6 kP(k) \\ &= e^{-\theta_1} + e^{-\theta_2} + \dots + e^{-\theta_6} \end{aligned}$$

and it can be easily shown that by NC's being a factor in the denominator of F assures that the mean increases as NC, the impact severity, increases. This is equivalent to the condition imposed on the mean of thoracic fractures.

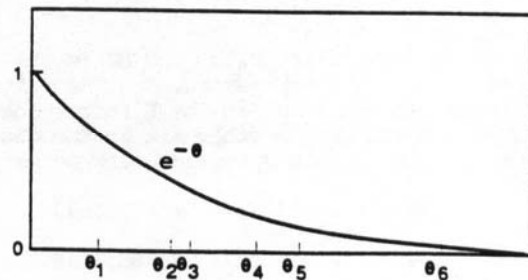


Figure 12: Graph of the function $e^{-\theta}$ in order to show how the location and the separation of the θ 's affect the AIS probabilities.

TABLE 2 — AIS INJURY DATA FOR BLUNT, FRONTAL IMPACTS
TO THE THORAX (7)

WEIGHT (LB)	AGE (YRS)	NORMALIZED COMPRESSION	AIS
167	81	0.444	5
117	80	0.393	4
145	78	0.418	4
156	19	0.375	2
125	29	0.350	1
164	72	0.417	4
179	65	0.425	4
119	65	0.395	4
139	75	0.185	0
149	54	0.194	0
164	51	0.459	6
130	64	0.447	4
164	52	0.346	4
119	61	0.321	1
141	64	0.315	3
208	46	0.310	1
169	75	0.257	2
175	66	0.269	3
110	76	0.363	4
138	72	0.371	2
138	67	0.420	4
126	76	0.435	6
134	58	0.428	4
89	52	0.310	2

THE PROBABILITY FOR AIS FOR BLUNT, FRONTAL IMPACTS TO THE THORAX

The data in Table 2 were analyzed to estimate the probability of injury assessed in terms of the AIS for blunt, frontal impacts to the thorax. Consult (7) for the experimental details.

The following model of F had the best figure of merit, i.e., the greatest likelihood function, of those evaluated;

$$F = W^{v_7} / [A (NC)^2]$$

where A is age, W is weight and NC is the normalized thoracic compression. The value of the single PML parameter in the F factor (designated v_7) and of the other six in the distribution of the previous section were determined to be

$$v_1=9.10095 \quad v_2=17.4476 \quad v_3=33.1090$$

$$v_4=23.9034 \quad v_5=285.478 \quad v_6=85.3498$$

$$v_7=-0.663331$$

Figure 13 shows, based on the above model of F, the probability distribution for a cadaver of age 65 years and of weight 160 pounds and normalized thoracic compression of 0.45. Note that the probabilities do not vary smoothly. Figure 14 shows two envelopes of the probabilities for weight of 160 pounds and normalized compression of 0.45; one envelope was for a cadaver of age 20 years and the other was for a cadaver of age 65 years. As to be expected, there is a shift to higher probabilities for the older cadaver.

From the distribution in Figure 13, one can compute the probability of an AIS less than or equal to any other AIS category by a summation of probabilities. However, a statistical procedure requiring fewer PML determined parameters and thus consuming fewer degrees of freedom is explained in the next section.

THE PROBABILITY DISTRIBUTION OF AIS<3 AND AIS>4

The AIS data in Table 2 were arbitrarily separated into two categories: AIS<3 and AIS>4. (Other pairs of categories could have been selected.) By this scheme, the probability distribution given by Eqs. (5) reduces to

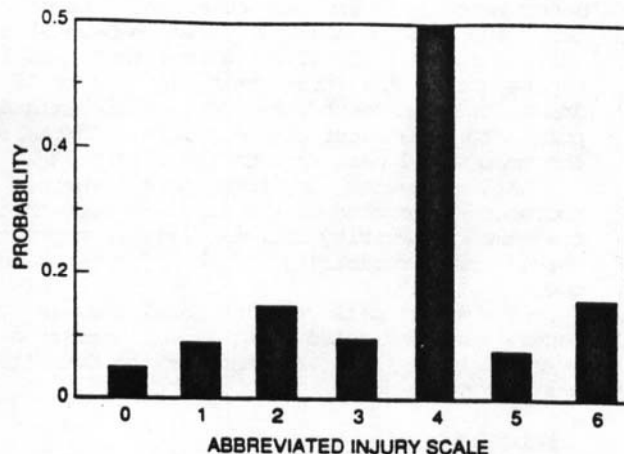


Figure 13: The AIS probability distribution for blunt thoracic impact of a cadaver for the following conditions: age 65 years, weight 160 lb and a normalized thoracic compression of 0.45.

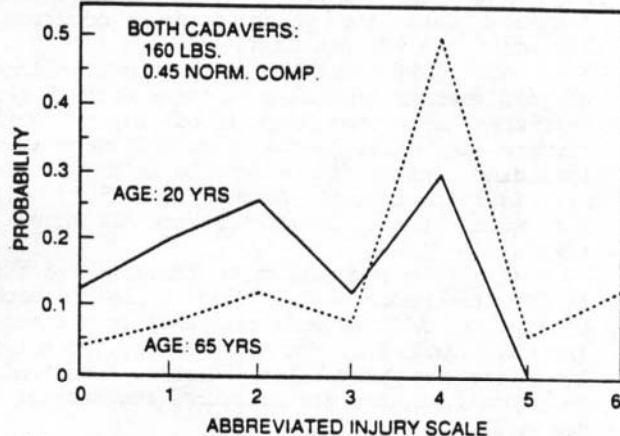


Figure 14: The AIS probability distributions for two cadavers of weight 160 lb and a normalized thoracic compression of 0.45; one cadaver of age 20 years and the other of age 65 years.

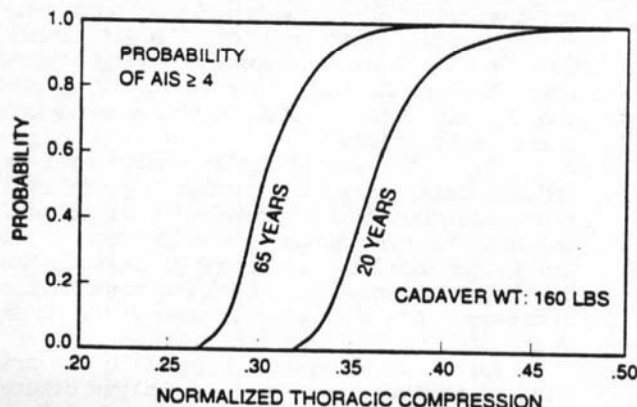


Figure 15: The probability of AIS₄ as a function of normalized thoracic compression for two cadavers of weight 160 lb; one cadaver of age 20 years and the other of age 65 years.

$$P(0) = 1 - e^{-\theta}$$

$$P(1) = e^{-\theta} \quad \text{Eqs. (7)}$$

where $P(0)$ is the probability of AIS₃, and $P(1)$ is the probability of AIS₄. This distribution has only one parameter instead of the six with AIS. The additional parameters that one might introduce would be expected to be fewer than the minimum of six required for the AIS.

The modeling for the single parameter θ was similar to that used for the six AIS parameters with Eqs. (6) reducing to the one equation

$$\theta = v_1 F$$

where v_1 was one PML parameter. Additional parameters were introduced through the factor F . The following model of F

$$F = W^2 / [A^3 (NC)^4]$$

had the highest figure of merit of those investigated on a trial and error basis. By the three additional v 's introduced through F , the model contained a total of four PML parameters. The values of the four v 's were

$$v_1 = 12.2122 \quad v_3 = 2.73002$$

$$v_2 = -2.59650 \quad v_4 = 18.1040$$

THE PROBABILITY OF AIS₄ FOR BLUNT, FRONTAL IMPACTS TO THE THORAX

Since there are only two possible outcomes, AIS₃ or AIS₄, the probability of AIS₄ also corresponds to the mean of the distribution of Eqs. (7). Also, one minus the probability of AIS₄ is equal to the probability of AIS₃.

Figure 15, based on the model of F in the preceding section, has plots of the probability of AIS₄ versus normalized thoracic compression for two cadavers each of weight of 160 pounds. One curve is for a cadaver of age 65 years and another for age 20 years. Both curves have a general "S" shape even though such a condition was not imposed by the analysis. As to be expected, the curve for a cadaver of 20 years is located to the right of a cadaver of age 65 years.

DISCUSSION

A test for which the outcome was no injury contains important information, and the data should be incorporated into the statistical analysis. In least squares one would have to be careful in including such data. Reference is made to fictitious fracture data in Figure 16. The solid line represents the true mean to be estimated by least squares. One would want to exclude the data points for no injury, because their use would introduce an error as indicated by the dashed line. Since the probability distribution for fractures used to implement the FML has zero fractures as an outcome, such data would not have to be excluded. Figure 17 contains fictitious data to illustrate a similar problem for AIS, except that the problem exists as both ends of the AIS scale. Since the probability distribution for AIS has

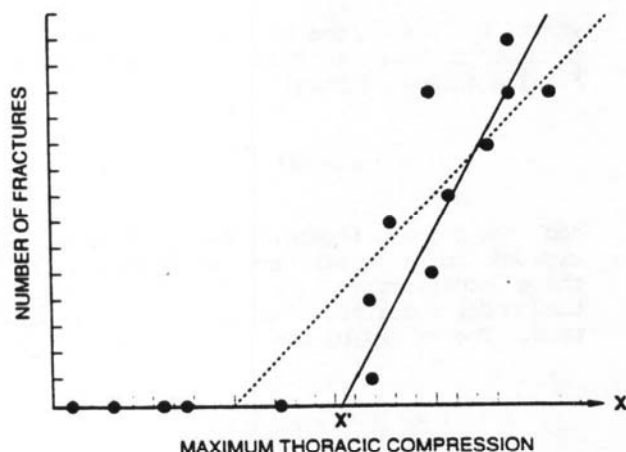


Figure 16: Fictitious fracture data to illustrate why a data sample containing tests for which no fractures occurred is not suitable for analysis by least squares.

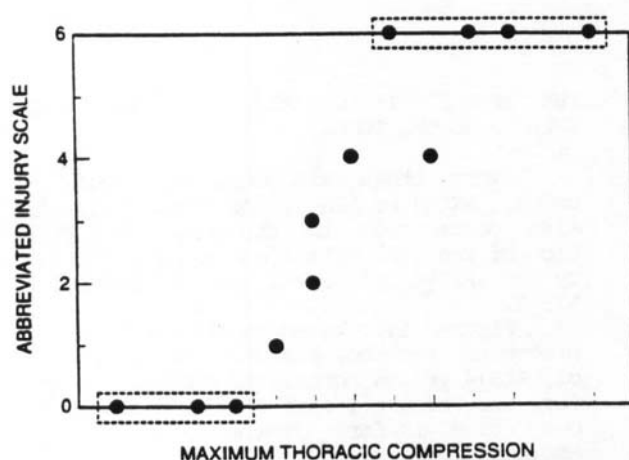


Figure 17: Fictitious AIS data which would not be suitable for analysis by least squares because of the data points (boxed in) at AIS=0 and AIS=6.

both zero and six as outcomes, data of both zero and six would not have to be excluded. (For purposes of illustration, the solid line representing the true mean in Figure 16 was drawn to intersect the horizontal axis at a point to the right of the origin. In reality, the mean would pass through the origin.)

All modeling in this paper, whether for thoracic fractures or the AIS, was such that as the impact severity decreased (i.e., approached zero) the probability of no injury approached one.

The two data samples analyzed in this paper were selected just to illustrate a new methodology. The adequacy of the data itself was not reviewed.

CONCLUSIONS

(1) Impact injury data are not observations from normal (Gaussian) probability distributions and, thus, they are not, in general, suitable for analysis by least squares. This conclusion is especially true if the sample contains data for which no injury occurred or for which the AIS equals six.

(2) The principle of maximum likelihood, as implemented in this report for both thoracic fractures and the Abbreviated Injury Scale, permits an entire data sample to be analyzed including tests for which no injury occurred or, in respect to the Abbreviated Injury Scale, for which there is one or more AIS scores of six.

(3) The probability of thoracic fractures is a phenomenon of nature, while the probability of AIS depends upon what is written in the AIS handbook. For this reason, two entirely different probability distributions had to be specified, one for fractures and another for the AIS.

RECOMMENDATIONS

(1) Unless an investigator has a solid reason for not doing so, all modeling relative to the principle of maximum likelihood, whether for thoracic fractures or the AIS, should be done so that as the impact severity approaches zero the probability for no injury approaches one. (In other words, if there is no impact, there is no injury.)

(2) In statistically analyzing a set of injury data, try to establish guidelines for the modeling of the probability distribution's parameters, as was done in this report. Clearly Recommendation (1) is one guideline. Another is requiring the mean of the distribution to increase as the impact severity increases. Search for other guidelines such as these.

(3) Investigate implementing the principle of maximum likelihood to analyze other sets of impact injury data such as for the lower limbs.

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DISCLAIMER

The views presented are those of the author and are not necessarily those of the National Highway Traffic Safety Administration, U. S. Department of Transportation.

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PAPER: Development of two Probability Distributions

SPEAKER: Arnold K. Johnson

Question: Guy Nusholtz, Chrysler

You're validating your model against some experimental data using cadavers. You make the assumption that if there is not impact there will be no injuries; no rib fractures. You could probably find some instances where you could set a cadaver up, put a seat-belt on him, have no impact, do an autopsy and find injury.

A. OK, I'm assuming you don't have biased cadavers. You've got a good point there, it wasn't covered in the distribution or in the paper.

Nusholtz: Obviously, in a vehicle you would expect that but probably you might want to consider that there might be some probability that at no tension you might get some sort of fractures.

A. That's a good point. I never thought of it. Thank you. I'll go back and give some thought to it.